Quantum sensing with utracold atoms interacting with bad cavities

XIII Escola de Física Roberto A. Salmeron

Philippe W. Courteille

Research team on interactions between Light & Matter

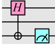
Organization of the talk

Quantum sensing sensing, with atoms, in cavities

Projection noise quantum sensing 2.0

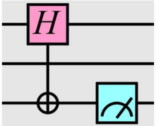
Synchronization of atomic dipoles in bad cavities $\rightarrow$ superradiance, spin-squeezing

Outlook entanglement witnesses, optically dense clouds

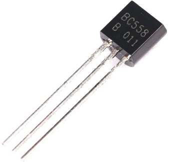


Quantum mechanics

and second generation quantum technologies



Quantum mechanics everywhere



Quantum revolution 1.0

transistor, nuclear energy, laser, atomic clocks, ...

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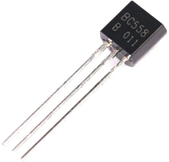

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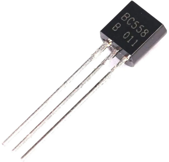

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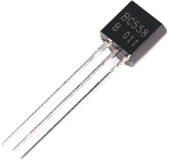

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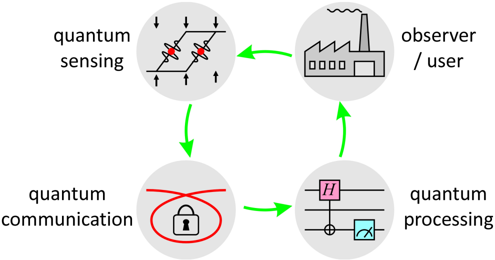

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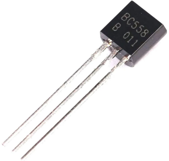

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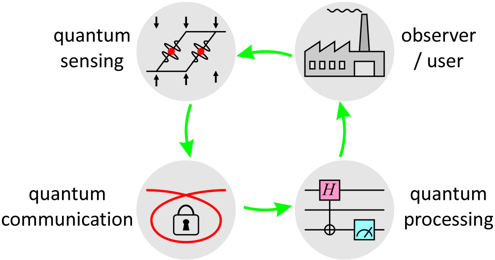

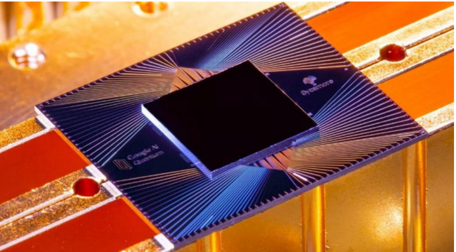

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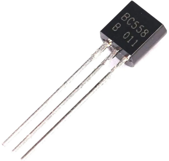

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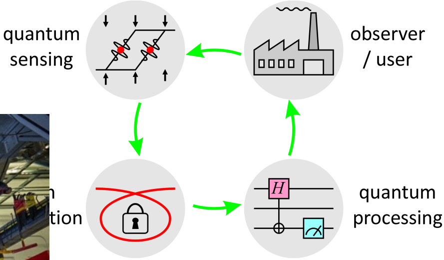

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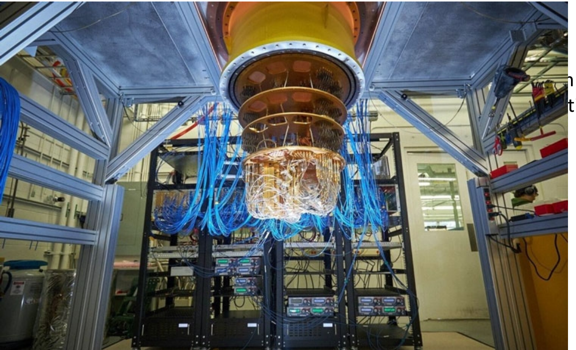

What is a sensor?

'device generating & providing information on events or changes in its environment' (Wikipedia)

time, gravitation, gravity gradients, accelerations and rotations, electric and magnetic fields, temperature, ...

Sensors everywhere

smartphone, 5G, autonomous driving, ...

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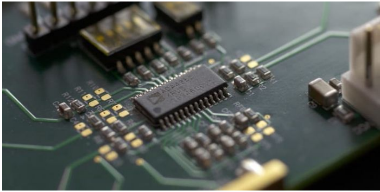
In fundamental science

measure weak forces with high sensitivity, strong forces with great accuracy!

What is a quantum sensor?

'measurement device exploiting quantum correlations in order to enhance sensitivity and resolution'

e.g. quantum superpositions or entanglement (Wikipedia)



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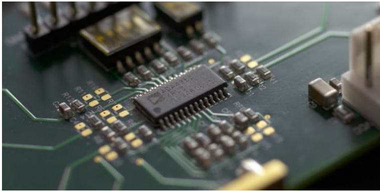
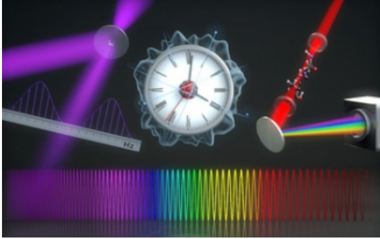


Advantages of quantum sensors

- precision & sensitivity
- speed
- robustness
- integrability, ...

What is an atomic quantum sensor?



atoms are 'quantum', some have ultra-narrow resonances

imprecision of best atomic clock: $2.5 \cdot 10^{-19} = 0.000\,000\,000\,000\,000\,000\,25$

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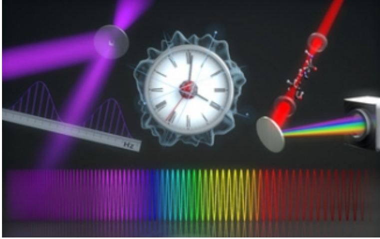

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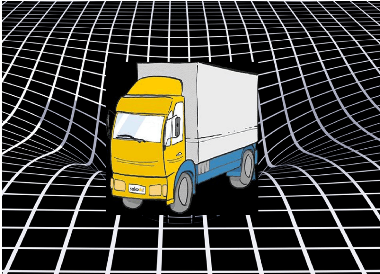
imprecision of best gravimeter: 10^{-9} $\longrightarrow$ measure deformation of gravity field caused by a truck

some projects in Brazil

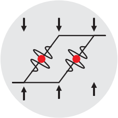


What is a quantum sensor 2.0?

Most current quantum sensors use single-atom quantum superpositions

today: individual atoms can be observed $\longrightarrow$ emergence of quantum jumps, ...

quantum sensor(qubit)



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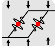

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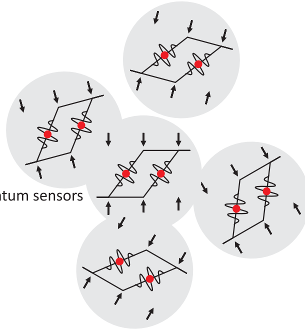
For good signal-to-noise $\longrightarrow$ *observe many atoms simultaneously*

Standard Quantum Limit / shot noise $(\propto \sqrt{N}^{-1})$



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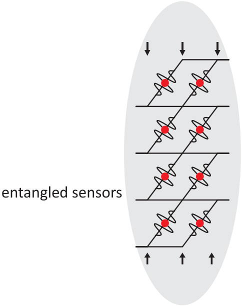
Entangled qubits allow precision beyond SQL (Nobel prize 2022)

Heisenberg limit $(\propto N^{-1})$ and beyond

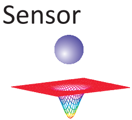
spin squeezing, squeezed light for gravitational wave detection



[Bouyer, Kasevich, PRA **96**, R1083 (1997) *Heisenberg-limited spectroscopy with degenerate Bose-gases*]

Quantum sensing using cavities?

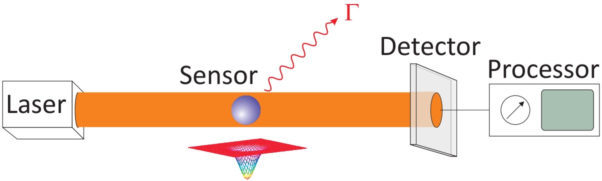
Atoms as sensor



Quantum sensing using cavities?

Atoms as sensor , *light as detector*

light interacts with the atoms and carries the information to the detector



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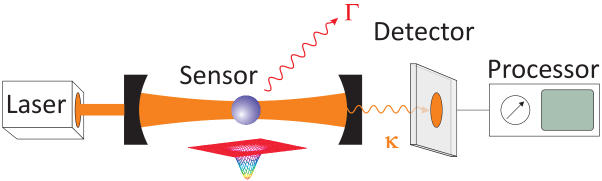

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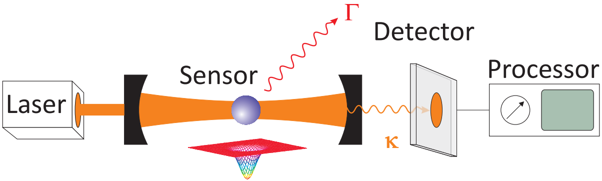
- isolate single light mode
- collective coupling of atoms $(g\sqrt{N} \gg \kappa)$ $\longrightarrow$ precondition for quantum correlation



Classical projection noise

Probability to be in |  ⟩ *or* |  ⟩

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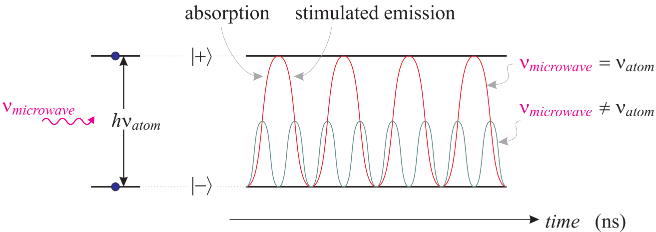
Probability to be in $| \quad + \quad \rangle + | \quad - \quad \rangle$



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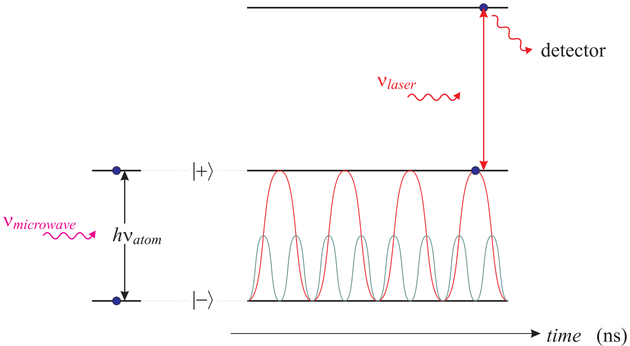

Projection noise in a two-level system

Probability to be in $|+\rangle$ *or* $|-\rangle$ $\qquad p_+$

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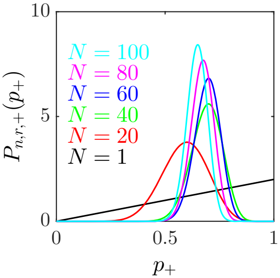

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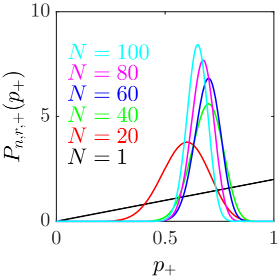
Expectation value and variance

$$\bar{r} = Np_+$$

$$(\Delta r)^2 = Np_+p_- \qquad \Longrightarrow \quad \frac{\bar{r}}{\Delta r} \propto \sqrt{N}$$



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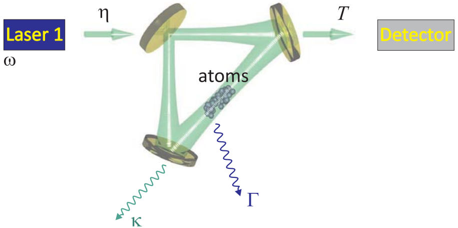

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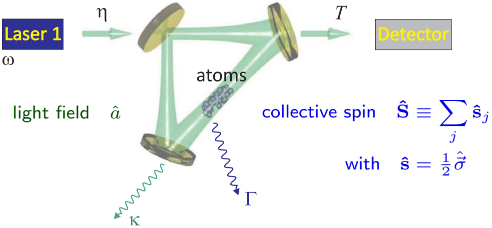
Laser 1
ω
η
T
Detector
atoms
light field $\hat{a}$
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κ
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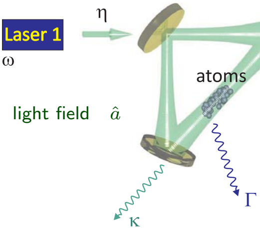

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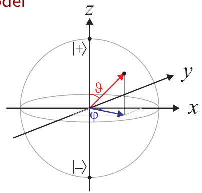
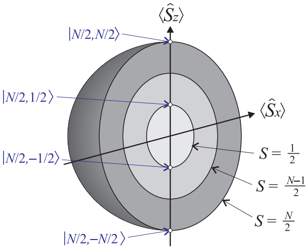
Atoms treated as non-interacting spins

no near field terms, only radiative coupling

coupled spin description $\Rightarrow$ Dicke model





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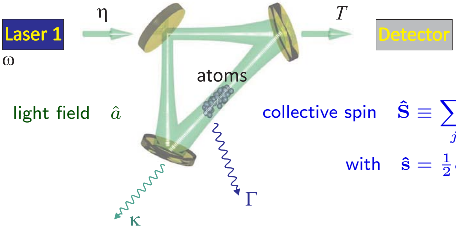

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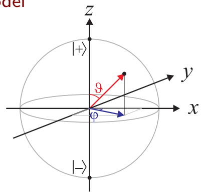
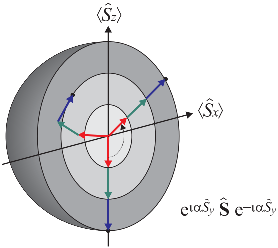
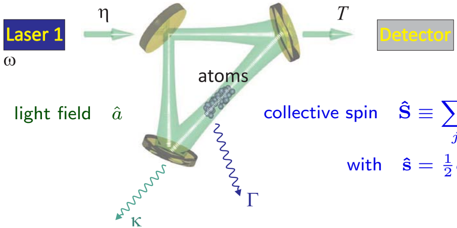




Linear terms $\hat{H} \propto \hat{S}_{x,y,z}$ *only perform rotations:* $e^{\imath\alpha\hat{S}_z}\; \hat{\mathbf{S}}\; e^{-\imath\alpha\hat{S}_z}$

$\Rightarrow$ a coherent spin state always remains a coherent spin state

Dicke model



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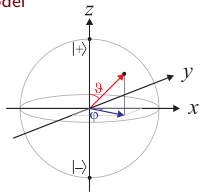

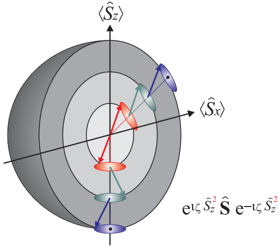

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$\Rightarrow$ a coherent spin state always remains a coherent spin state

$\Rightarrow$ no entanglement can be generated by linear spin operators in the Hamiltonian

Spin-squeezing requires non-linear terms: $e^{\imath\zeta\hat{S}_z^2}\;\hat{\mathbf{S}}\;e^{-\imath\zeta\hat{S}_z^2}$

Quantum noise in continuous variable representation

Glauber state $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ with $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

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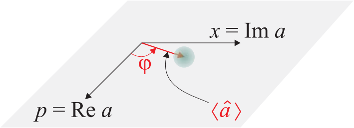

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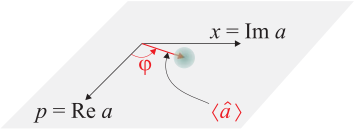

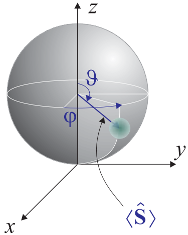

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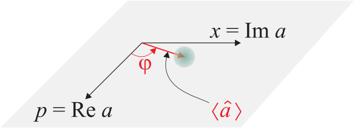
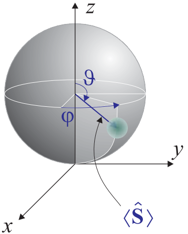
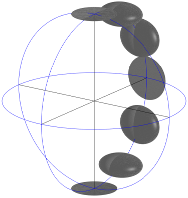
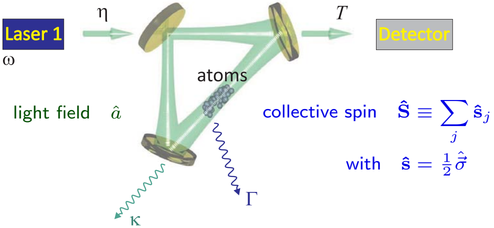






Why bad cavities?



[Norcia, Lewis-Swan, Cline, Bihui Zhu, Rey, Thompson, Science **361**, 259 (2018)]
[Salvi, Poli, Vuletić, Tino, PRL 120, 033601 (2018)]
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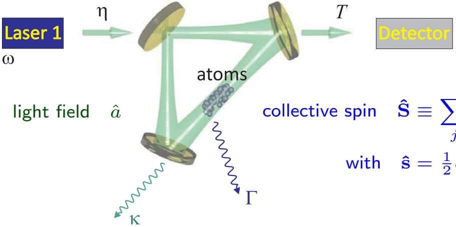

collective spin $\hat{\mathbf{S}} \equiv \sum_j \hat{\mathbf{s}}_j$

with $\hat{\mathbf{s}} = \frac{1}{2}\hat{\vec{\sigma}}$

resonant Dicke model Hamiltonian $\hat{H} = -\imath\eta(\hat{a} - \hat{a}^\dagger) + g(\hat{S}_+\hat{a} + \hat{a}^\dagger\hat{S}_-)$

[Norcia, Lewis-Swan, Cline, Bihui Zhu, Rey, Thompson, Science **361**, 259 (2018)]
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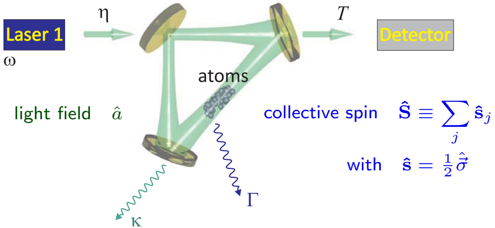

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Bad-cavity limit: $\kappa \gg \Gamma$ $\Longrightarrow$ *adiabatic slaving of cavity dynamics* $\Longrightarrow$ *eliminate* $\hat{a}$ *from Hamiltonian*

[Norcia, Lewis-Swan, Cline, Bihui Zhu, Rey, Thompson, Science **361**, 259 (2018)]

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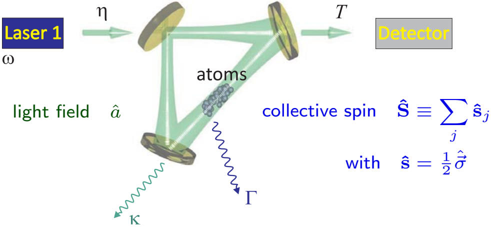

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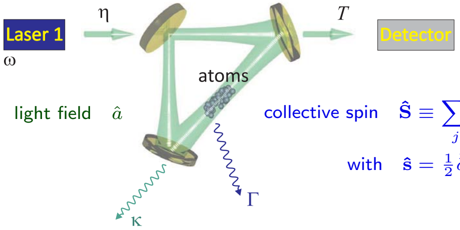
Why bad cavities?



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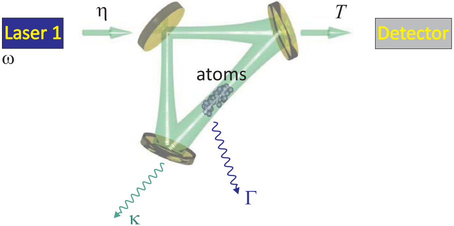
Bad-cavity limit: $\kappa \gg \Gamma$ $\Longrightarrow$ *adiabatic slaving of cavity dynamics* $\Longrightarrow$ *eliminate* $\hat{a}$ *from Hamiltonian*

approximated Hamiltonian $\hat{H} \simeq U\ \hat{S}_+\hat{S}_- \quad \simeq U\ \hat{S}_z^2$

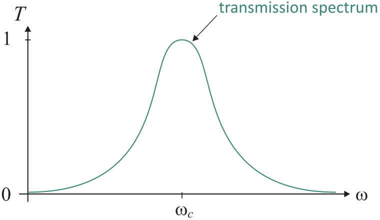
$\Longrightarrow$ QM is linear! Non-linearity is always an approximation!

$\Longrightarrow$ non-linearity can generate entanglement, spin squeezing and superradiant lasing

Storyboard for an experiment



1) set up an experiment in the 'bad' cavity parameter regime ($\kappa \to \infty$)



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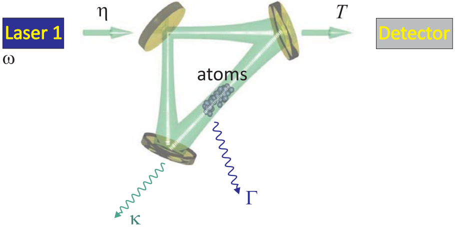
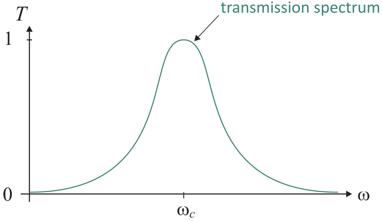
Storyboard for an experiment



1) set up an experiment in the 'bad' cavity parameter regime ($\kappa \to \infty$)

2) take atoms with narrow transitions ($\Gamma \to 0$) and cool them



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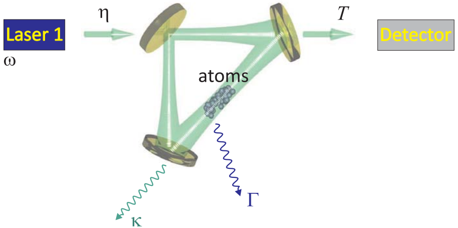

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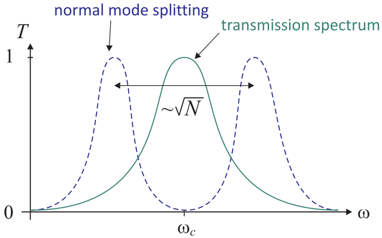

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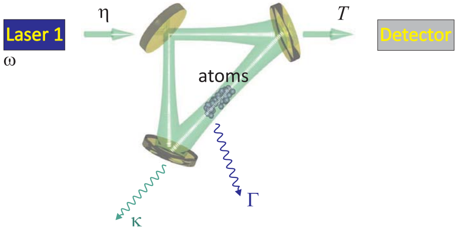

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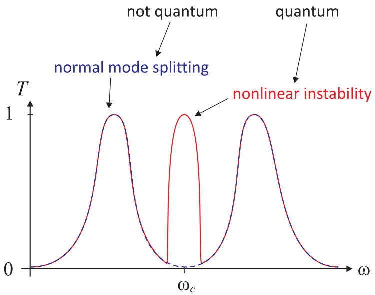
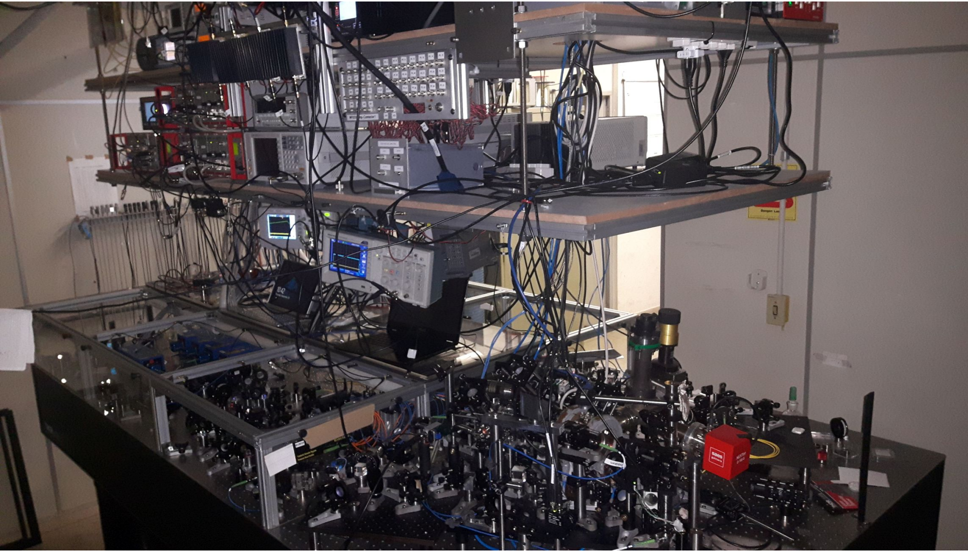
4) verify non-linearity 'on-resonance' ($\Delta_c = 0$)

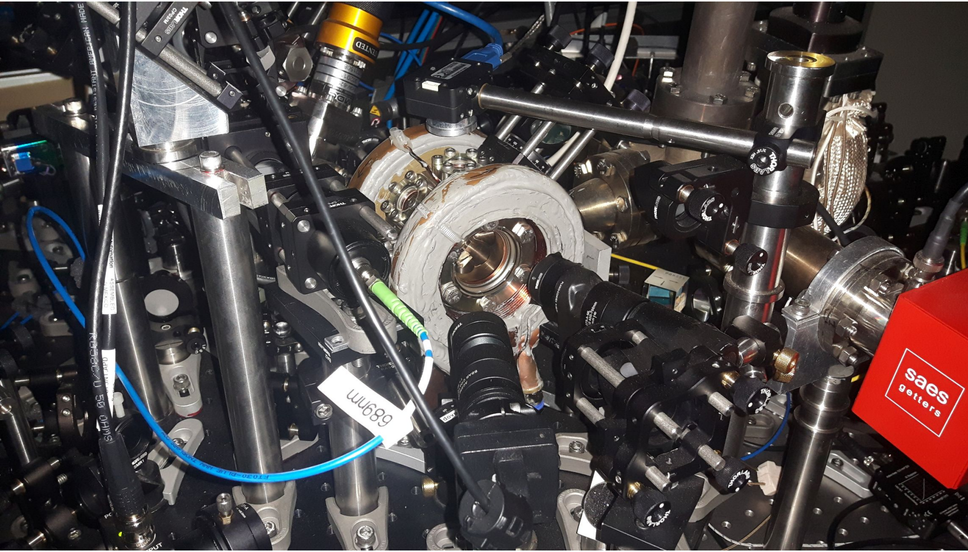


[Rivero, de França, Pessoa, Teixeira, Slama, Courteille, New J. Phys. **25**, 093053 (2023)]

The experiment

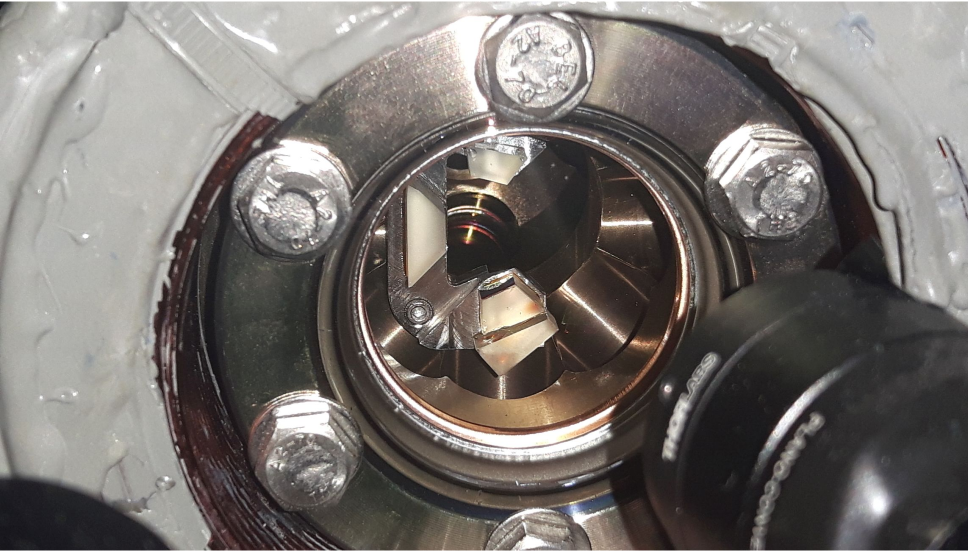


The experiment



The experiment

strontium $\Gamma = 7.5\,\mathrm{kHz}$ cavity decay $\kappa = 4.3\,\mathrm{MHz}$

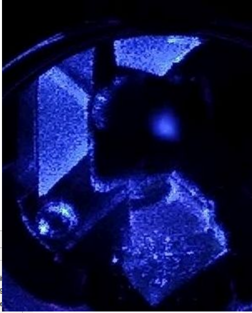


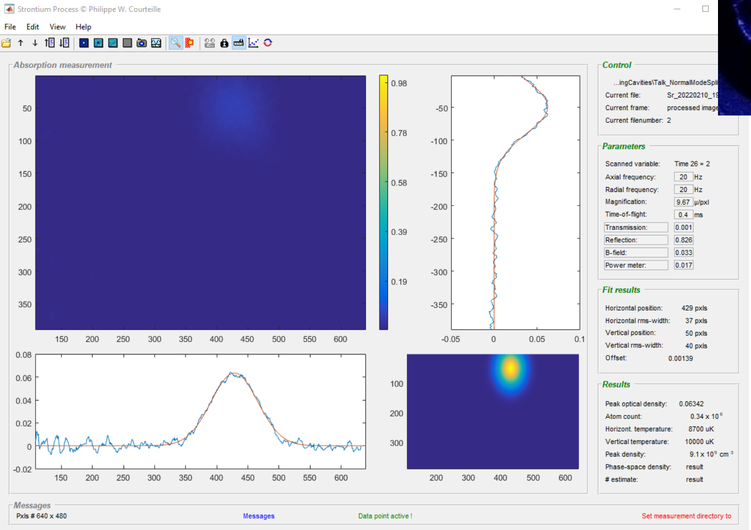
Experimental procedure & state of the art

experimental control

Experimental procedure & state of the art

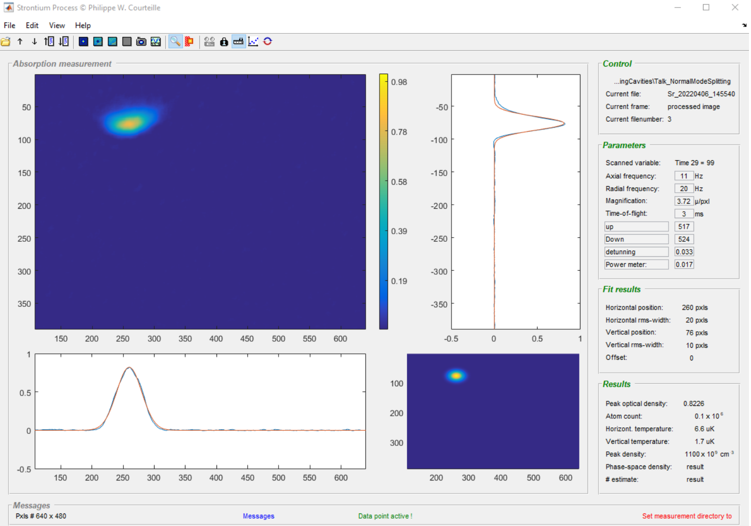
trapping atoms in the blue MOT: $N = 10^6 \qquad T = 5\,\mathrm{mK}$





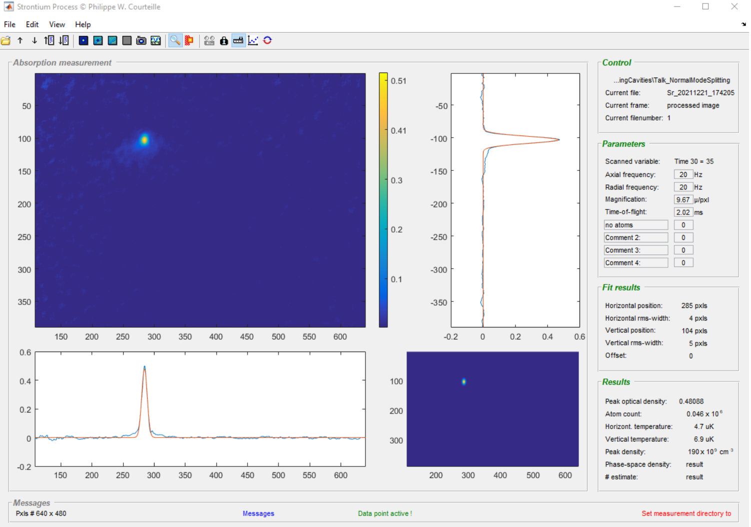
Experimental procedure & state of the art

cooling atoms in the red MOT



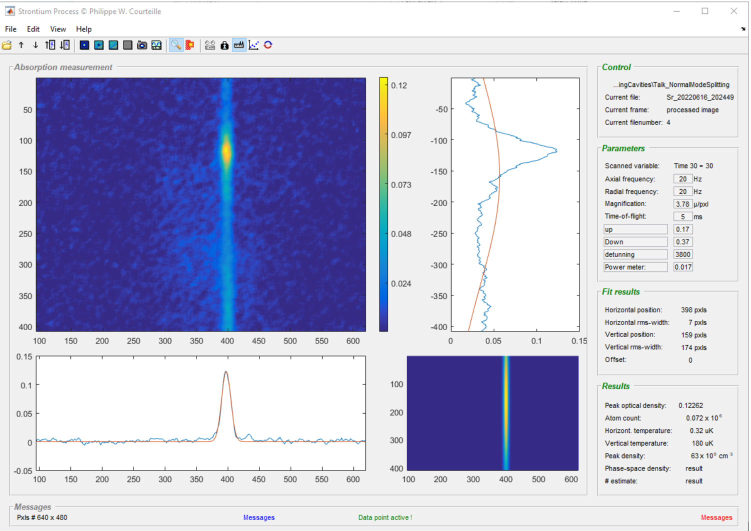
Experimental procedure & state of the art

cooling atoms in the red MOT: $N = 2 \cdot 10^5 \qquad T = 1\,\mu\text{K}$



Experimental procedure & state of the art

transferring atoms to the ring cavity mode via magnetic field ramp



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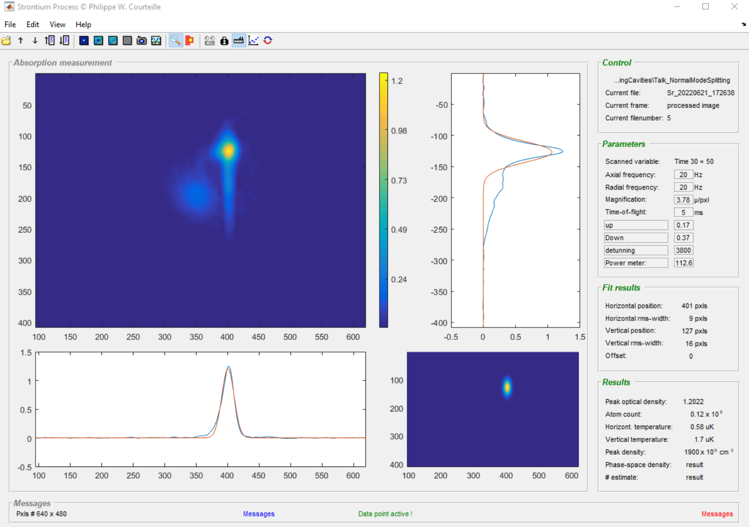
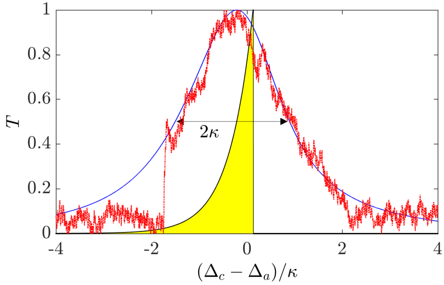
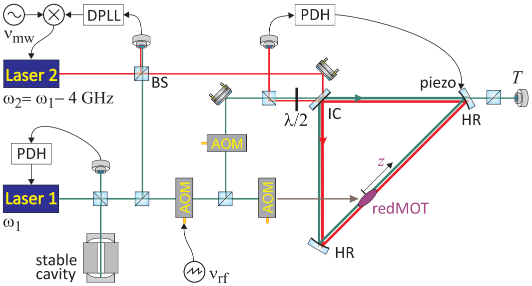


Normal mode splitting

scanning laser frequency which pumps the cavity





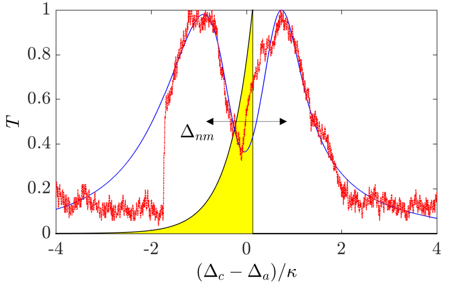
[Rivero, Beli, Armijo, da Silva, Kessler, Shiozaki, Teixeira, Courteille, Appl. Phys. B **128**, 44 (2022)]

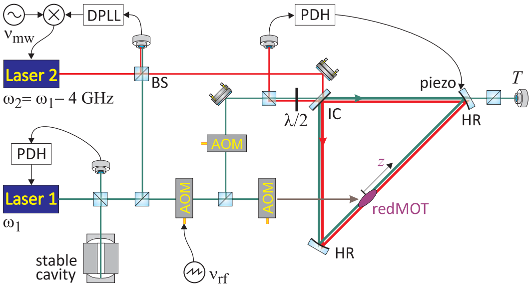
[Rivero, de França, Pessoa, Teixeira, Slama, Courteille, New J. Phys. **25**, 093053 (2023)]

Normal mode splitting

scanning laser frequency which pumps the cavity

$$\Gamma \ll \kappa \ll g\sqrt{N} \equiv \Delta_{nm}$$





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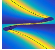
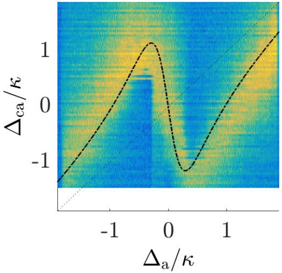
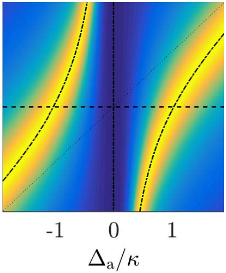
Normal mode splitting $\equiv$ 1D photonic band gap



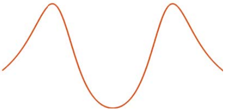
avoided crossing $+$ instable feature





$$\Delta_{ca} \equiv \Delta_a - \Delta_c$$

$$\Delta_c = \frac{Ng^2\Delta_a}{\Delta_a^2 + \Gamma^2/4}$$



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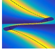

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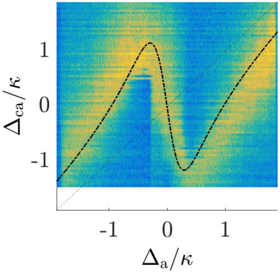

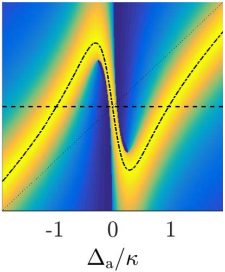

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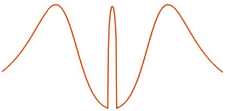

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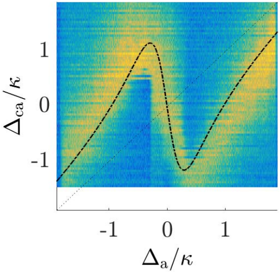

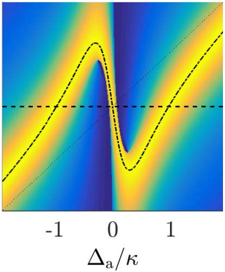

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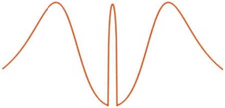
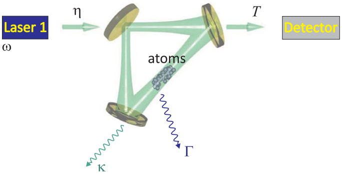


adiabatic elimination only near $\Delta_a = 0$

Dissipative spin-squeezing

$\Delta_a = 0 = \Delta_c \quad \Rightarrow \quad$ non-linearity provided by **collective dissipation + pumping** rather than Hamiltonian



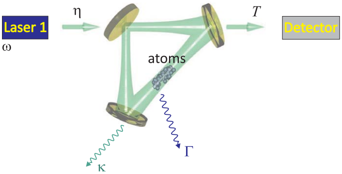
[Somech, Shahmoon et al., PRA **108**, 0203725 (2023)]

[Somech and Shahmoon, PRX Quantum **5**, 010349 (2024)]

Dissipative spin-squeezing

$\Delta_{\mathsf{a}} = 0 = \Delta_{\mathsf{c}} \quad \Rightarrow \quad$ non-linearity provided by **collective dissipation + pumping** rather than Hamiltonian

$\hat{S}_- |S, \alpha\rangle \simeq \alpha |S, \alpha\rangle \quad \& \quad [\hat{S}_-, \hat{H}_{\text{eff}}] = 0 \quad \Leftrightarrow \quad 0 = \frac{d}{dt}\hat{\rho} = \frac{d}{dt}|S, \alpha\rangle\langle S, \alpha|$



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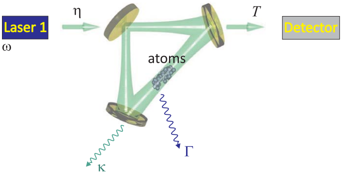
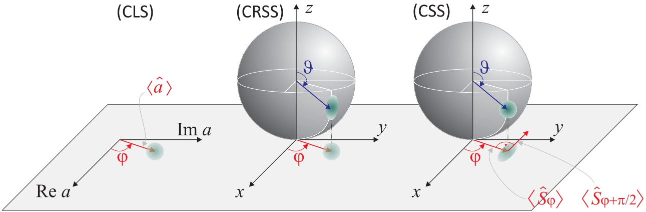


Coherently Radiating Spin-Squeezed state (CRSS)



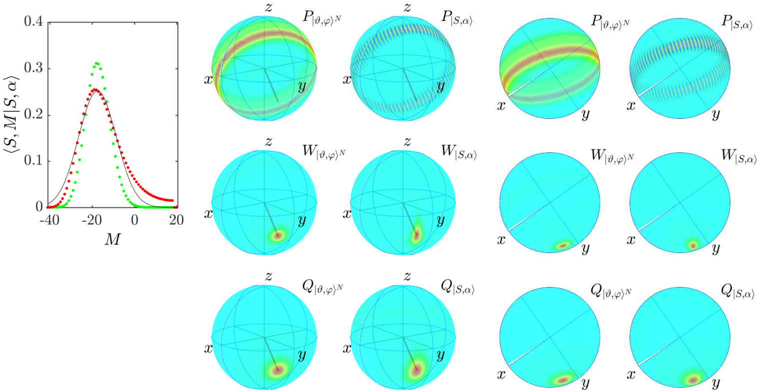
light scattered by a collective spin: $\hat{a}^\dagger = \hat{a}_0^\dagger + G\hat{S}_-$

[Somech, Shahmoon et al., PRA **108**, 0203725 (2023)]

[Somech and Shahmoon, PRX Quantum **5**, 010349 (2024)]

Coherently radiating spin-squeezed states

P-, Wigner, and Q-function of CSS and CRSS



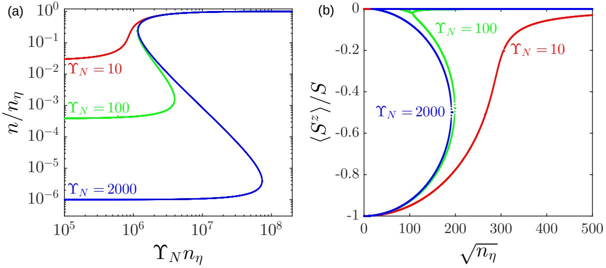
[Somech, Shahmoon et al., PRA **108**, 0203725 (2023)]

[Somech and Shahmoon, PRX Quantum **5**, 010349 (2024)]

Characterization of CRSS

quantum jumps between two bistable states: a CSS and a CRSS at steady state

cavity transmission — atomic inversion



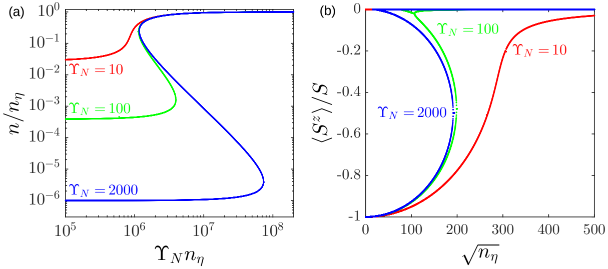
scaled photon number — field amplitude

[Leppenen and Shahmoon, arXiv:2404.02134]

Characterization of CRSS

quantum jumps between two bistable states: a CSS and a CRSS at steady state

cavity transmission | atomic inversion

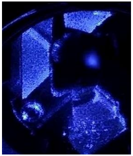


scaled photon number | field amplitude

cooperatividade $\Upsilon_N = \dfrac{Ng^2}{\kappa\Gamma} = \dfrac{N\kappa_c}{\Gamma} = \dfrac{\text{collective decay}}{\text{single-atom decay}}$

[Leppenen and Shahmoon, arXiv:2404.02134]

Quintessence



Done:

- bistability observed on resonance with a 'bad cavity'! $\Longrightarrow$ non-linearity

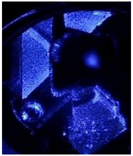
[Meiser et al., PRL **102**, 163601 (2009)]

[Debnath, Zhang, Mølmer, PRA **98**, 063837 (2018)]

[Rosario, Santos, Piovella, Kaiser, Cidrim, R. Bachelard, PRL 133, 050203 (2024)]

[recent work of groups of Vuletic, Schleier-Smith, Thompson, Rey, ...]

Quintessence

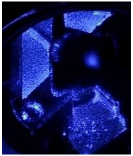


Done:

- bistability observed on resonance with a 'bad cavity'! $\Longrightarrow$ non-linearity
- large atomic saturation achieved on resonance! $\Longrightarrow$ dynamics intrinsically 'quantum'

[Meiser et al., PRL **102**, 163601 (2009)]
[Debnath, Zhang, Mølmer, PRA **98**, 063837 (2018)]
[Rosario, Santos, Piovella, Kaiser, Cidrim, R. Bachelard, PRL 133, 050203 (2024)]
[recent work of groups of Vuletic, Schleier-Smith, Thompson, Rey, ...]

Quintessence



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- large atomic saturation achieved on resonance! $\Longrightarrow$ dynamics intrinsically 'quantum'

non-linearity + quantumness $\Longrightarrow$ implementation of new ideas on squeezing or superradiant lasing?

[Meiser et al., PRL **102**, 163601 (2009)]

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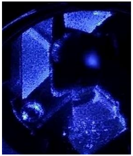

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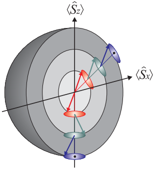

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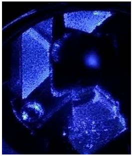

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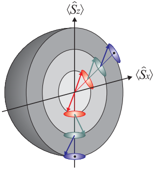
To do:

find optical spin-squeezing witnesses

generate inversion $> 50\%$ (e.g. via optical pumping) for light amplification



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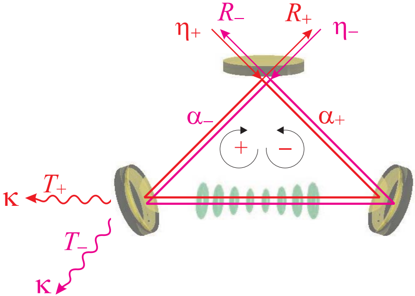
Outlook on: Optically dense clouds in optical lattices

atom numbers $N \approx 200000$

lattice sites $N_s \approx 300$

optical density $OD = \frac{6N}{k^2 w^2} \approx 3$



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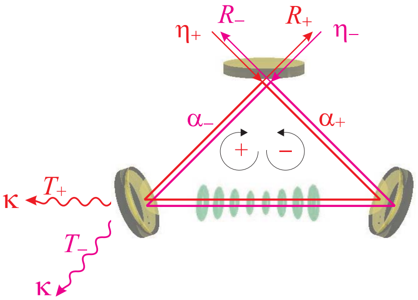
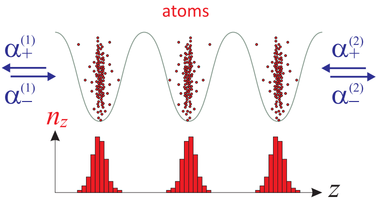


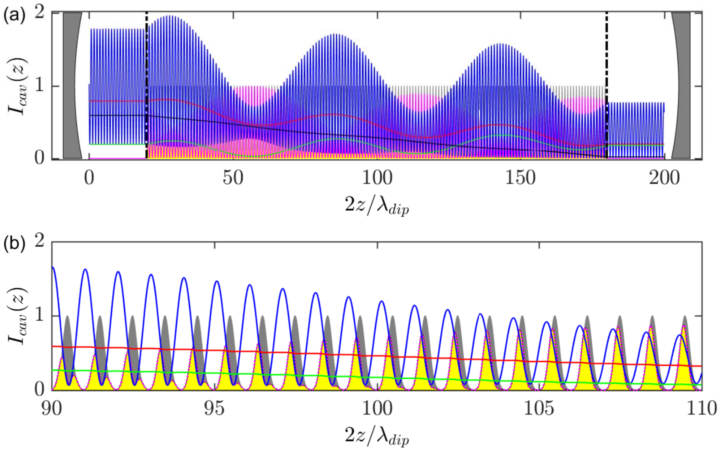
$\Longrightarrow$ absorption and multiple scattering

$\Longrightarrow$ Open Dicke Model (ODM) no longer valid

$\Longrightarrow$ use Transfer Matrix Model (TMM)



Intracavity intensity



[Deutsch, Spreeuw, Rolston, Phillips, PRA **52**, 1394 (1995)]

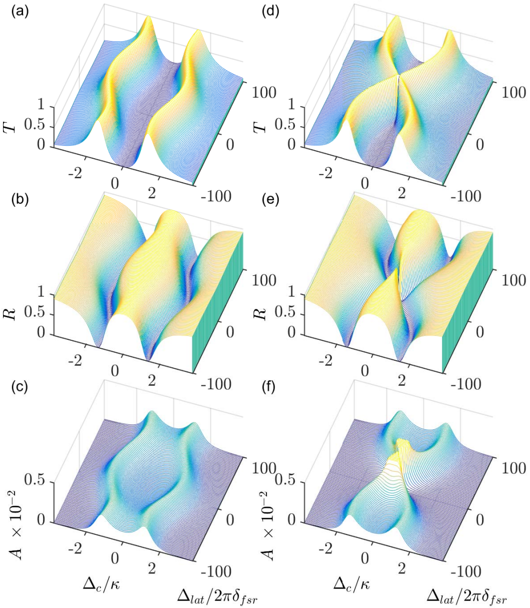
[Slama, von Cube, Kohler, Zimmermann, Courteille, Phys. Rev. A **73**, 023424 (2006)]

[Schilke, Zimmermann, Courteille, Guerin, Nature Phot. **6**, 101 letter (2012)]

[Samoylova, Piovella, R. Bachelard, Ph.W. Courteille, Opt. Comm. **312**, 94 (2014)]

[Rivero, de França, Pessoa, Teixeira, Slama, Courteille, New J. Phys. **25**, 093053 (2023)]

Normal mode splitting $\equiv$ 1D photonic band gap



linear cavity

$OD < 1 \;\Rightarrow\;$ ODM = TMM

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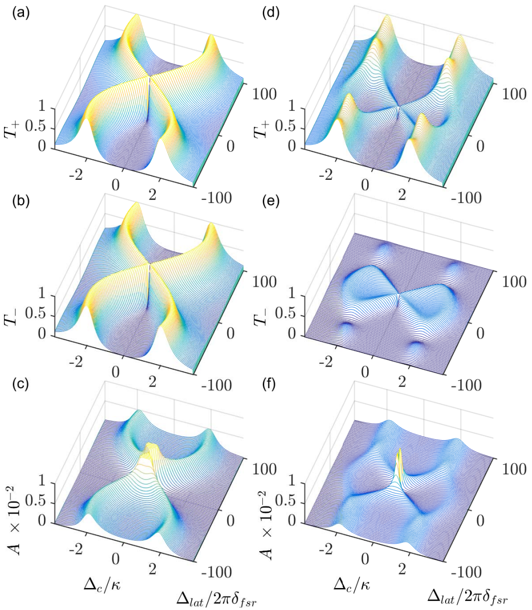

ring cavity

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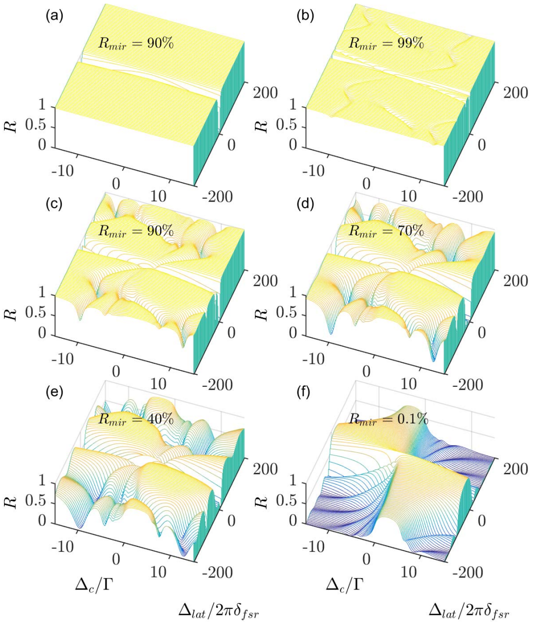

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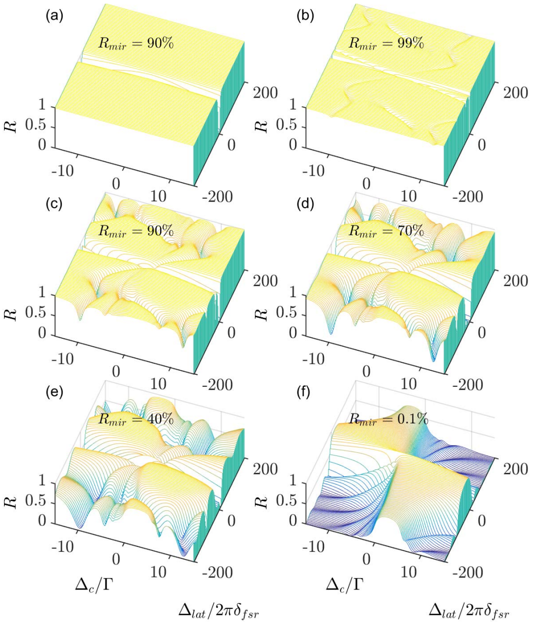


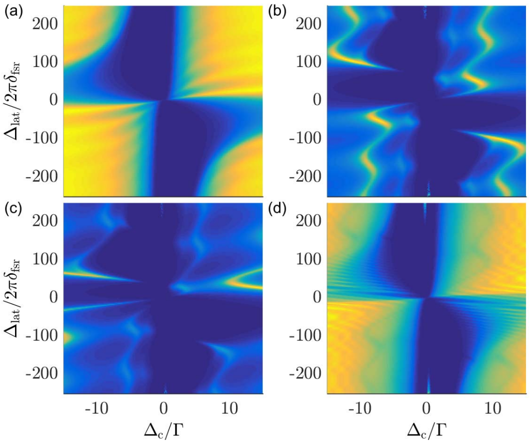
linear cavity

$OD > 1 \;\Rightarrow\;$ ODM $\neq$ TMM

$\Rightarrow$ appearance of photonic bandgap

$\Rightarrow$ notion of cavity mode function fails

Dense cloud with macroscopic boundary conditions



cavity = filter for specific reflection paths

Toward a full quantum model

search for a model working at any $'I_{\mathsf{sat}}'$ and $'OD'$

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need to consider impact of cooperative environment

$$\hat{H} = \hbar \sum_{\mathbf{k},\lambda} \sum_j (\hat{\sigma}_j^+ + \hat{\sigma}_j^-)(g_{\mathbf{k}\lambda} \hat{a}_{\mathbf{k}\lambda} + g^*_{\mathbf{k}\lambda} \hat{a}^\dagger_{\mathbf{k}\lambda})$$

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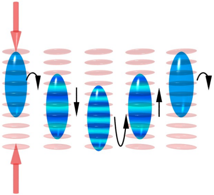
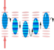
$g_{\mathbf{k}\lambda}$ shaped by cavity

Matter wave Bloch oscillations

for inertial sensing

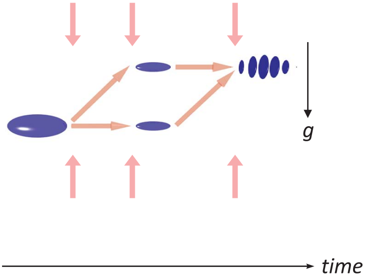


Gravimetry with Bose-Einstein condensates



differential phase shift of de Broglie waves

$\longrightarrow$ matter wave interferometers



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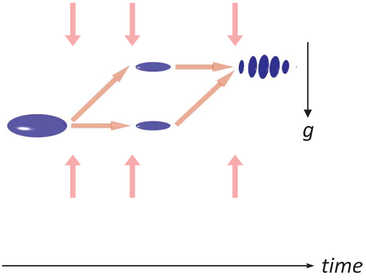
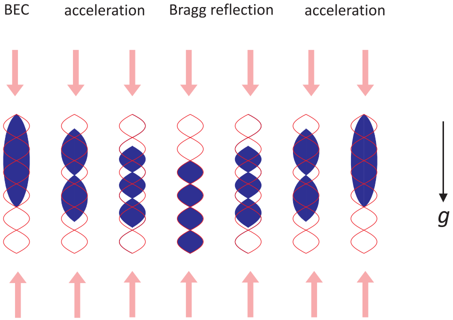


matter wave Bloch oscillations in a periodic potential

- wavelength $\lambda_{dB} = \frac{h}{mv}$



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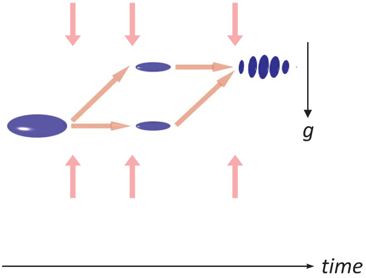
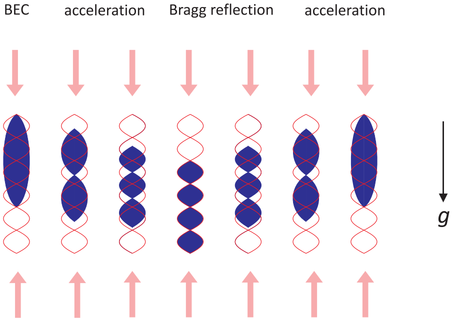


matter wave Bloch oscillations in a periodic potential

- wavelength $\lambda_{dB} = \frac{h}{mv}$
- frequency $\nu_b = \frac{mg}{2\hbar k}$

$\longrightarrow$ measure gravity g



[Ben Dahan, Peik, Castin, Salomon, PRL **76**, 4508, (1996)]

Brazilian gravimeter

República Federativa do Brasil
Ministério da Indústria, Comércio Exterior e Serviços
Instituto Nacional da Propriedade Industrial

(21) **BR 102015007944-3 A2**

(22) Data do Depósito: 09/04/2015

(43) Data da Publicação: 07/11/2017

(54) Título: DISPOSITIVO E MÉTODO PARA MEDIDA DA ACELERAÇÃO GRAVITACIONAL

(51) Int. Cl.: G01V 7/00

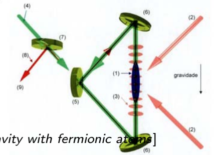
(52) CPC: G01V 7/00

(73) Titular(es): UNIVERSIDADE DE SÃO PAULO - USP

(72) Inventor(es): PHILIPPE WILHELM COURTEILLE; ROMAIN PIERRE MARCEL BACHELARD

(74) Procurador(es): MARIA APARECIDA DE SOUZA

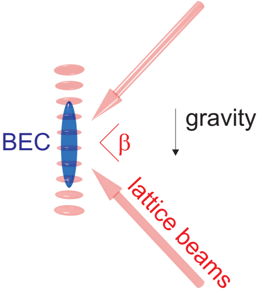
(57) Resumo: DISPOSITIVO E MÉTODO PARA MEDIDA DA ACELERAÇÃO GRAVITACIONAL. A presente invenção refere-se a dispositivo e método, em especial a um gravímetro baseado em interferometria atômica, no qual os átomos são resfriados até uma temperatura em que formam uma onda de matéria coerente e depois são transferidos dentro de uma onda estacionária vertical de luz quase-ressonante com uma transição atômica. Os átomos são colocados dentro de uma cavidade óptica anular (23) bombeada em uma direção por um feixe laser (18) e executam oscilações de Bloch, cuja frequência é estritamente proporcional à aceleração gravitacional. O método compreende as etapas de: a) Preparar um feixe de átomos frios; b) Capturar este feixe atômico por uma armadilha magneto-óptica operada numa transição atômica larga e resfriá-lo para temperaturas em torno de 5 mK; c) Resfriar os átomos ainda mais por uma armadilha magneto-óptica operada numa transição atômica fina para temperaturas em torno de 300 nK; d) Transferir a onda de matéria para onda estacionária de luz; e) Incitar os átomos a executar oscilações de Bloch devido à aceleração gravitacional; f) Injetar um las(...)



[Inguscio et al., WO 2005 076042A1 (2005), *Apparatus and method for the measurement of the acceleration of gravity with fermionic atoms*]

[Courteille et al., BR 10 2015 007944-3 (2015), *Device and method for gravitational acceleration measurement*]

Continuous monitoring Bloch oscillations in a cavity



[Samoylova, Piovella, Robb, Bachelard, Courteille, Opt. Exp. **23**, 14823 (2015)]

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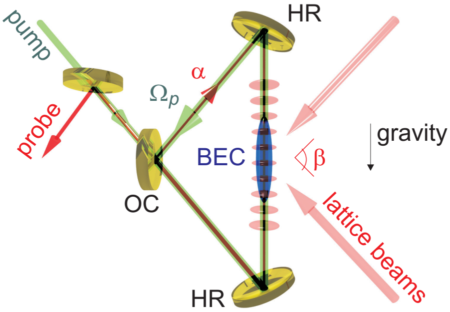

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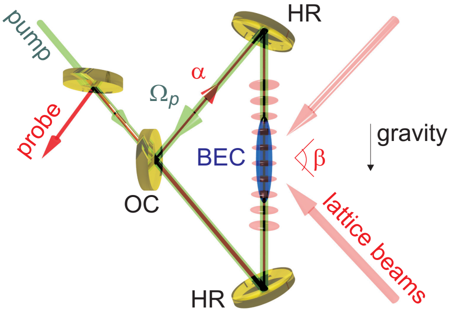

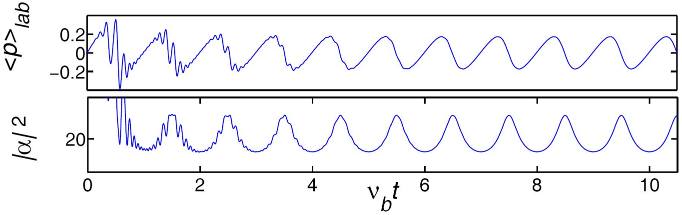

$N = 8 \cdot 10^4$

$\kappa = 160\omega_r$

$U_0 = 0.04\omega_r$

$W_0 = 80U_0$

[Samoylova, Piovella, Robb, Bachelard, Courteille, Opt. Exp. **23**, 14823 (2015)]

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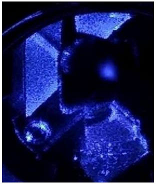

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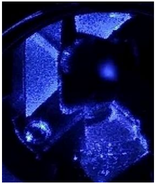

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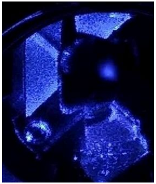
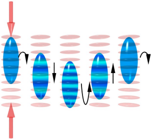


To do:

search for signatures of Bloch oscillations in light modes



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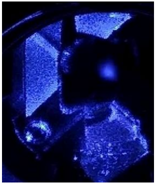

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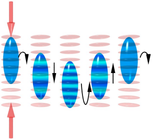

search for signatures of Bloch oscillations in light modes

continuous monitoring of gravity

The team

Raul Teixeira, Dalila Rivero, Gustavo de França, Claudio Pessoa

Ana Cipris, Mayerlin Nuñez Portela

Scripts available

FAPESP

CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

inct institutos nacionais de ciência e tecnologia

Model: Bloch band spectrum

Schrödinger equation for a particle at rest in a standing light wave (periodic 1D potential)

after that adiabatic elimination of internal states

$$\hat{H}\hat{\psi} = -\frac{\hbar^2}{2m}\frac{\partial^2\hat{\psi}}{\partial x^2} + \frac{\hbar W_0}{2}\sin(2k_l x)\hat{\psi}$$

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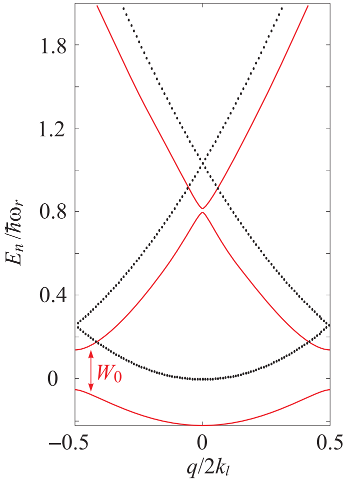
confine q to 1. Brillouin zone & calculate eigenenergy spectrum



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moving particle in standing light wave

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time-dependent solution with $\nu_b = \frac{g}{\omega_r}$ and $\omega_r = \frac{\hbar k_l^2}{2m}$

$$\frac{dc_n}{dt} = -4\imath\omega_r(n \qquad)^2 c_n + \frac{W_0}{2}(c_{n+1} - c_{n-1})$$

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external force

expand into plane (Bloch) waves (de Bloch) with $|c_n(t)|^2$ momentum states populations

$$\hat{\psi}(x,t) = \sum_{n=-\infty}^{\infty} c_n(t)e^{2\imath nk_l x} \cdot e^{\imath mgxt/\hbar}$$

transform into moving frame

time-dependent solution with $\nu_b = \frac{g}{\omega_r}$ and $\omega_r = \frac{\hbar k_l^2}{2m}$

$$\frac{dc_n}{dt} = -4\imath\omega_r(n + \nu_b t)^2 c_n + \frac{W_0}{2}(c_{n+1} - c_{n-1})$$

[Samoylova, Piovella, Hunter, Robb, Bachelard, Courteille, Las.Phys.Lett. **11**, 126005 (2014)]
[Samoylova, Piovella, Robb, Bachelard, Courteille, Opt.Exp. **23**, 14823 (2015)]

Model: Dynamics

CONCLUSION

moving particle in standing light wave

$$\imath\hbar\frac{\partial\hat{\psi}}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\hat{\psi}}{\partial x^2} + \frac{\hbar W_0}{2}\sin(2k_l x)\hat{\psi} \quad - mgx\hat{\psi}$$

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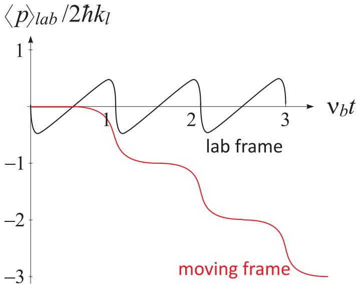
center-of-mass momentum

$$\langle p\rangle_{lab} = \sum_n n|c_n(t)|^2 + \nu_b t$$



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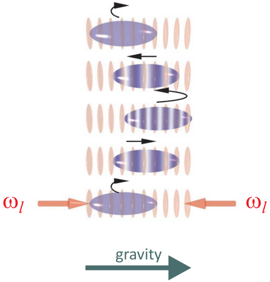
2 pictures of Bloch oscillations



a. Bragg reflection of matter waves by a standing light wave

when $\lambda_{dB} = \frac{h}{p} \stackrel{!}{=} \frac{2\pi}{k_l} = \lambda_l$

$\Longleftrightarrow \quad p = \hbar k_l$ recoil of 1 photon

$\Longleftrightarrow$ edge of Brillouin zone

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b. Light-matter interaction needs an atomic transition

$\Longrightarrow$ Raman transition between counterprop. momentum states

with photonic recoil $2\hbar k_l$